

Density and expansion effects on pion spectra in relativistic heavy-ion collisions

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Abstract

We compute the pion inclusive momentum distribution in heavy-ion collisions at AGS energies, assuming thermal equilibrium and accounting for density and expansion effects at the time of decoupling. We compare to data on mid rapidity charged pions produced in central Au + Au collisions and find a very good agreement. The shape of the distribution at low $m_t - m$ is explained in part as an effect arising from the high mean pion density achieved in these reactions. The difference between the positive and negative pion distributions in the same region is attributed in part to the different average yields of each kind of charged pions.

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A great deal of experimental effort has been devoted in recent years to the production of a highly compressed state of matter in high-energy collisions between heavy ions. It has been speculated that at sufficiently high baryonic densities or temperatures, a phase transition from hadronic matter to a quark-gluon plasma can occur. The study of particle spectra should provide useful information about the dynamics and evolution of the kind of matter formed in these reactions.

Some of the most copiously produced particles in central high-energy heavy ion collisions are pions. An understanding of pion production during these collisions has long been searched, particularly in view of one of the most remarkable properties exhibited by their spectra [1], commonly referred to as an enhancement of either the low or the high transverse momentum regions in the inclusive single pion distribution, as compared to p-p collisions. This property is concomitant with the difficulty to describe the invariant transverse mass distribution with a single exponential function [2].

Several possibilities have been put forward to explain the peculiar shape of the distributions, amongst them, the different contributions to the pion yield coming from the decay of Δ resonances produced during different stages of the collision [3] and the superposition of primary pions and pions coming from resonance decay, mainly Δ 's [4]. The importance of transverse flow in describing the spectra has also been stressed [5]. The proper treatment of Coulomb final state interactions has also been pointed out. More recently, Wong and Mostafa [6] noticed that when particles feel the effects of a boundary during the evolution from the first stages of the collision to their final free streaming, their momentum distribution is affected due to the discretization of energy levels and the correspondingly different density of states introduced by the finite size of the system just before freeze out. This idea is based on the concept of a pion liquid, first discussed by Shuryak [7] and was further developed in Ref. [8] with the introduction of a finite chemical potential associated to the mean pion multiplicity per event in central collisions.

Calculations based on the effects of a boundary resort to the assumption of thermal equilibration. They are successful in reproducing the concave shape of the distribution at high transverse momentum but fail to describe its overall fall off. This failure could have been anticipated since, as pointed out in Ref. [9], thermodynamics is tantamount of hydrodynamics and a simple thermal spectrum should be corrected by the Doppler shift resulting from collective expansion. A challenge remained as to how to incorporate hydrodynamical effects into a description of the transverse momentum distribution based on discrete energy states. In this work we meet this challenge and lay down the formalism required to account simultaneously for expansion and boundary effects in a phenomenological calculation of pion spectra in relativistic heavy-ion collisions. We apply this formalism to compute the invariant transverse momentum distribution for mid rapidity pions at AGS energies. We find a very good agreement with data on central Au+Au reactions at 11.6 A GeV/c. Further distortions of the spectra arising from Coulomb effects will be discussed in an upcoming work.

In order to collect the necessary ingredients for the calculation, recall that if the average pion separation d ever becomes smaller than the average range of the pion strong interaction d_s (~ 1.4 fm) during the evolution of the collision, the pion dispersion curve can be modified and the collective properties of the pion system resemble those of a liquid rather than those of a gas [7]. An important consequence is the appearance of a surface tension that acts as a reflecting boundary. The E-802/866 collaboration has reported that a baryon density

of about eight times normal nuclear density is achieved in central Au+Au collisions at 11.6 A GeV/c. A large fraction of this density is due to pions. From Fig. 3 of Ref. [2], one can read that the total number of charged pions one unit around central rapidity in this kind of reactions is about 200. Under the assumption that the number of neutral pions in the same rapidity interval is half the total number of charged ones, the total pion yield in central collisions at mid rapidity is $dN_\pi/dy \sim 300$. Since the average pion separation is inversely proportional to one third of the pion density $n_\pi = (1/At_0)dN_\pi/dy$, where $A(\sim 64 \text{ fm}^2)$ is the transverse area of the reaction and $t_0(\sim 1 \text{ fm})$ is a typical formation time [10], $d \sim 0.6 \text{ fm} < d_s$ and thus the condition to regard the pion system as a liquid is met.

Pions that move towards the boundary of the system feel the attractive potential behind them and are reflected back. The reflection details depend on the wave length of the given particle but the property introduced by the reflecting surface is that it allows very little wave function *leakage* and to a good approximation, the pion wave functions vanish outside the boundary. When, as a consequence of the expansion of the initially compressed and hot system, the pion average separation becomes larger than the range of strong interactions, the system becomes a free gas but the transition between the liquid and the gas phases is very rapid and the pion momentum distribution should be determined by the distribution just before freeze out.

Thus the system of pions can be considered as confined and their wave functions as satisfying a given condition at the boundary just before freeze out. In this case, the energy states form a discrete set. The shape of the volume within the confining boundary deserves some attention. For reactions with a large degree of transparency, Bjorken like geometry, with a predominantly longitudinal elongation, seems better suited. However, for AGS energies, a significant amount of stopping has been reported by the E-802/866 collaboration in central Au+Au reactions [2]. In this case, a more symmetric geometry between transverse and longitudinal directions seems appropriate. We thus consider a scenario in which the system of pions of a given species is in thermal equilibrium and is confined within a sphere of radius R (fireball) as viewed from the center of mass of the colliding nuclei at the time of decoupling. As discussed in Ref. [11], this time needs not be the same over the entire reaction volume. Nevertheless, in the spirit of the fireball model of Ref. [12], we consider that decoupling takes place over a constant time surface in space-time. This assumption should be essentially correct if the freeze out interval is short compared to the system's life time.

In the absence of expansion, the solution has been found in Ref. [8]. This involves solving the Klein-Gordon equation to find the stationary wave functions satisfying

$$\left[\frac{\partial^2}{\partial t^2} - \nabla^2 + m^2 \right] \psi(\mathbf{r}, t) = 0, \quad (1)$$

subject to the condition

$$\psi(|\mathbf{r}| = R, t) = 0, \quad (2)$$

and finite at the origin. In order to incorporate the effects of a hydrodynamical flow, we observe that the presence of an ordered motion, represented by a four-velocity field $u^\mu = \gamma(r)(1, \mathbf{v}(r))$, amounts to a redistribution of momentum in each of the fluid cells, as viewed from a given reference frame (the center of mass in our case). The tendency of matter to

occupy a larger volume is compensated by the distribution of momenta in each cell becoming narrower [12]. The distribution in the cell becomes also centered around the momentum associated with the velocity of the fluid element. Consequently, the thermal spectrum in each cell should be described on top of this collective flow, that is, referred from the collective fluid's element momentum. To describe this behavior of the expanding, bound system of pions, we make the substitution of the momentum operator p^μ by $p^\mu - mu^\mu$, where m is the pion mass. The term mu^μ represents the collective momentum of the given pion fluid element. The corresponding equation becomes

$$\left[-\left(i\frac{\partial}{\partial t} - m\gamma(r)\right)^2 + \left(-i\nabla - m\gamma(r)\mathbf{v}(r)\right)^2 + m^2 \right] \psi(\mathbf{r}, t) = 0 \quad (3)$$

and we look for stationary solutions subject to the same condition in Eq. (2) and also finite at the origin. We consider a parametrization of the three velocity vector $\mathbf{v}(r)$ that scales with the distance from the center of the fireball.

$$\mathbf{v}(r) = \beta \frac{r}{R} \hat{\mathbf{r}}, \quad (4)$$

We identify this velocity with the transverse flow velocity and ignore any asymmetry between transverse and longitudinal expansion. $0 < \beta < 1$, represents the surface fireball velocity. The corresponding explicit expression for $\gamma(r)$ is

$$\gamma(r) = \frac{1}{\sqrt{1 - \beta^2 r^2 / R^2}}. \quad (5)$$

Eq. (3) with the gamma factor given by Eq. (5) can only be solved numerically. In order to provide an analytical solution, we approximate the function γ by the first terms of its Taylor expansion

$$\gamma(r) \simeq 1 + \frac{\beta^2}{2} \frac{r^2}{R^2}. \quad (6)$$

This approximation is valid for not too large values of β . In this case, Eq. (3) becomes an equation for particles moving in a spherical harmonic well with a rigid boundary.

The stationary states are

$$\begin{aligned} \psi_{nlm'}(\mathbf{r}, t) &= \frac{A_{nl}}{\sqrt{2E_{nl}}} e^{-iE_{nl}t} e^{im\beta r^2/(2R)} Y_{lm'}(\hat{\mathbf{r}}) \\ &\times e^{-\alpha_{nl}^2 r^2/2} r^l {}_1F_1\left(\frac{(l+3/2)}{2} - \frac{\varepsilon_{nl}^2}{4\alpha_{nl}^2}, l+3/2; \alpha_{nl}^2 r^2\right), \end{aligned} \quad (7)$$

where ${}_1F_1$ is a confluent hypergeometric function and $Y_{lm'}$ is a spherical harmonic. The quantities A_{nl} are the normalization constants and are found from the condition

$$\int d^3r \psi_{nlm'}^*(\mathbf{r}, t) \overleftrightarrow{\frac{\partial}{\partial t}} \psi_{nlm'}(\mathbf{r}, t) = 1. \quad (8)$$

The parameters α_{nl} and ε_{nl} are related to the energy eigenvalues E_{nl} by

$$\begin{aligned}\alpha_{nl}^4 &= m(E_{nl} - m)\beta^2/R^2, \\ \varepsilon_{nl}^2 &= E_{nl}(E_{nl} - 2m).\end{aligned}\tag{9}$$

E_{nl} are given as the solutions to

$${}_1F_1\left(\frac{(l+3/2)}{2} - \frac{\varepsilon_{nl}^2}{4\alpha_{nl}^2}, l+3/2; \alpha_{nl}^2 R^2\right) = 0.\tag{10}$$

The normalized contribution to the momentum distribution from the energy state with quantum numbers n, l, m' is given in terms of the absolute value squared of the Fourier transform of Eq. (7), namely,

$$\psi_{nlm'}(\mathbf{p}) = \int d^3r e^{-i\mathbf{p}\cdot\mathbf{r}} \psi_{nlm'}(\mathbf{r}).\tag{11}$$

Since the problem has an azimuthal symmetry, the wave function in momentum space does not depend on the quantum number m' and is a function only of the momentum magnitude.

$$\psi_{nlm'}(\mathbf{p}) = \psi_{nlm'}(p)\delta_{m'0}.\tag{12}$$

Consequently, the momentum distribution is obtained by weighing the contribution from each state with the statistical Bose–Einstein factor and adding up the contribution from all of the states

$$\frac{d^3N}{d^3p} = \sum_{n,l} \frac{\phi_{nl}(p)}{e^{(E_{nl}-\mu)/T} - 1}\tag{13}$$

where $\phi_{nl}(p)$ is defined by

$$\phi_{nl}(p) = \frac{2E_{nl}}{(2\pi)^3} |\delta_{m'0}\psi_{nlm'}(p)|^2\tag{14}$$

and the chemical potential μ is computed from

$$N = \sum_{n,l} \frac{(2l+1)}{e^{(E_{nl}-\mu)/T} - 1},\tag{15}$$

for a given number of particles N . Eq. (15) follows from Eq. (13) after integration over d^3p .

Fig. 1 shows the systematics obtained by varying the parameters involved. The curves in Figs. 1a, b and c are computed for mid rapidity pions, $y = 0$, for which the assumption of spherical expansion should not be important since these do not experience the effect of longitudinal flow. Fig. 1a shows the behavior of the distribution for a fixed temperature $T = 120$ MeV, a fixed value of the surface expansion velocity $\beta = 0.5$ and a total number of particles $N = 150$ for various values of the fireball's radius R . Notice the convex shape of the distribution at low $m_t - m$ for large values of R and the transition to a concave shape with decreasing values of R . This is a density effect since for large R and a fixed total number of particles the density is lower than for smaller values of R . In the former case, the value taken by the chemical potential is far from the first energy state, whereas in the latter, the chemical potential is close to the energy of this state and thus the lowest lying energy states

contribute with a more significant statistical weight. The same effect can be obtained by keeping a fixed radius and varying the number of particles.

Fig. 1b shows the behavior of the distribution when varying the temperature T maintaining fixed values of the radius $R = 8$ fm, the surface expansion velocity $\beta = 0.5$ and the total number of particles $N = 150$. As could be expected, the main effect goes into the effective slopes describing the distribution's overall fall off. The distribution for $T = 100$ MeV rises more steeply at low values of $m_t - m$, due to the proximity of the system to the critical temperature for Bose-Einstein condensation, as discussed in Ref. [8]. Fig. 1c shows the effect of varying the surface expansion velocity comparing the cases with $\beta = 0$ and $\beta = 0.5$ keeping fixed the values of the radius $R = 8$ fm, the temperature $T = 120$ MeV and the total number of particles $N = 150$. Notice that the effect corresponds also to assigning different overall effective inverse slopes to each curve, the largest one corresponding to the curve with non-vanishing expansion velocity. Finally, Fig. 1d shows the shape of the distribution for a rapidity away from the central region, in this case with the parameters $T = 120$ MeV, $\beta = 0.5$, $R = 8$ fm and $N = 150$ for $y_{lab} = 3.0$. Notice the pronounced bending upwards of the distribution at large values of $m_t - m$. This effect can be attributed to a larger density of states at high energy eigenvalues as compared to a calculation without boundary.

With these systematics at hand, we proceed to describe the mid rapidity pion data on central Au+Au reactions at 11.6 A GeV/c. [2]. Data correspond to pions within a rapidity interval $|\Delta y| < 1$ around central rapidity. Given our assumption of a spherically symmetric fireball, our calculation will compare best to this part of the spectrum since these are the pions that do not experience the effects of a longitudinal flow that might be different from the flow in the transverse direction.

Rather than performing an exhaustive search in the whole parameter space and in order to test the plausibility of this type of description, here we fix the value of the parameters involved to reasonable and more or less accepted values. It is clear that a more complete analysis requires the proper treatment of Coulomb effects, this will be the subject of an upcoming work. We take [13] $T = 120$ MeV, $\beta = 0.5$ (corresponding to an average collective expansion velocity $\langle v \rangle \simeq 0.4c$). For the mean negative pion multiplicity we take $N_{\pi^-} = 160$ [14]. We consider a fireball radius $R = 8$ fm. Fig. 2a shows the theoretical distribution compared to data. In order to compare with the invariant differential cross section reported in Ref. [2] which is normalized to a subset of (central rapidity) 116 negative pions, the curve has been multiplied by a constant $\mathcal{N} = 0.56$ that minimizes the χ^2 when we compare to data above $m_t - m = 0.4$ GeV, the region where Coulomb effects should start becoming less significant. Above $m_t - m = 0.4$ GeV the agreement between data and theory is very good. Below $m_t - m = 0.4$ GeV the curve follows the shape of the data points but these last are still above the calculation. This could be a good feature since one knows that the long-range Coulomb effects should push the distribution for low momentum negative pions upwards, given that their Coulomb interaction with the overall positive charge is attractive.

We now use these parameters to describe data on positive pions. Fig. 2b shows the theoretical distribution calculated for $T = 120$ MeV, $\beta = 0.5$, $R = 8$ fm but a total positive pion multiplicity $N_{\pi^+} = 115$ [14], compared to data. In order to compare with the invariant differential cross section reported in Ref. [2] which is normalized to a subset of (central rapidity) 94 positive pions, the curve has been multiplied by the constant $\mathcal{N} = 0.59$ that

minimizes the χ^2 when we compare to data above $m_t - m = 0.4$ GeV. The agreement between data and theory is also very good for the region above $m_t - m = 0.4$ GeV. However, the raise of the curve below $m_t - m = 0.4$ GeV is less steep than for the negative pion case. This is easy to understand since for positive pions the density is lower than for the negative ones. Also in the same region, the theoretical curve is marginally below data and the Coulomb distortion will push it even more below. We speculate that this signals that data prefer a slightly lower value of the radius but again, this can only be confirmed after the proper inclusion of Coulomb effects.

In conclusion, we have shown that a very good description of mid rapidity, charged pion spectra can be achieved by a phenomenological calculation whose key ingredient is the proper treatment of the large pion density produced in central, relativistic Au+Au reactions. This large density leads to consider the pion system as confined during the early stages of the collision, before decoupling. Such scheme has consequences on both ends of the spectra. At low transverse mass, the convex shape of the distribution is due to the large value of the chemical potential associated with the mean pion multiplicity per event. At high transverse momentum, the convex shape of the distribution is due to the higher density of states as compared to a calculation without boundary. Another important element is the inclusion of collective flow. We find that an average flow velocity of $\langle v \rangle \simeq 0.4c$ together with a temperature $T = 120$ MeV does a very good job describing data above $m_t - m = 0.4$ GeV when the fireball's radius is about $R = 8$ fm. A more conclusive statement can be made only after we include a proper treatment of the Coulomb corrections [15]. Perhaps more importantly is the fact that the different rates at which the positive and negative pion distributions rise at low values of $m_t - m$ can be understood in part as an effect related to the correspondingly different measured yields, making the negative pion subsystem denser than the positive pion one.

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FIGURE CAPTIONS

Fig. 1: Systematics obtained by varying (a) the fireball's radius R , (b) the temperature T and (c) the surface expansion velocity β for rapidity $y_{cm} = 0$. (d) Distribution for a rapidity $y_{lab} = 3.0$.

Fig. 2: (a) Theoretical distribution $(2\pi m_t)^{-1} d^2 N / dm_t dy$ computed with the parameters $T = 120$ MeV, $\beta = 0.5$, $N_{\pi^-} = 160$ and $y_{cm} = 0$ compared to data from the E-802/866 on mid rapidity negative pions from central Au+Au reactions at 11.6A GeV/c. The theoretical curve has been multiplied by the constant $\mathcal{N} = 0.56$ that minimizes the χ^2 when compared to data above $m_t - m = 0.4$ GeV. (b) Distribution computed with the same parameters but

with $N_{\pi^+} = 115$ compared to data on mid rapidity positive pions from the same reaction. The theoretical curve has been multiplied by the constant $\mathcal{N} = 0.59$ that minimizes the χ^2 when compared to data above $m_t - m = 0.4$ GeV. Data are $(2\pi m_t \sigma_{trig})^{-1} d^2\sigma/dm_t dy$ in the rapidity interval $0 < \Delta y < 0.2$. The total measured yield spans the rapidity interval $|\Delta y| < 1$ around central rapidity [2].





